Symmetry and the order of events in time. The asymmetrical order of events in time in a reversible energy converter

Ben-Zion GINZBURG[1]**, Randy WAYNE**[2]
[1]*Plant Biophysical Laboratory. Plant Science Department*
The Silberman Institute of Life Sciences
The Hebrew University of Jerusalem. Givat Ram,
Jerusalem 91904, ISRAEL
[2]*Laboratory of Natural Philosophy, Department of Plant Biology, Cornell University,*
Ithaca, NY 14853 USA
e-mail: row1@cornell.edu

Abstract

The positive increase of entropy that occurs in irreversible processes has always served as a criterion for distinguishing the past from the future. The unerring ability of the Second Law of Thermodynamics to predict the order of events in time based on a change of entropy in macroscopic systems has led to the concept of the thermodynamical arrow of time. In this paper we analyze a cyclical reversible energy converter, in which thermal energy is converted into useful work without any net change of entropy. While it is usually assumed that time is an outsider in the equations of reversible thermodynamics[1, 2, 3], if not an illusion itself[4], it has been clear to many theoretical physicists from Laplace to Einstein that even in theory, all events must be considered to take place in time[5, 6, 7, 8, 9]. Our analysis of a reversible thermodynamic system shows that there is, in theory, an asymmetry in the events occurring in time. This leads us to the hypothesis that there may be another determinant, more fundamental than the increase of entropy in irreversible processes, for establishing an alternative thermodynamic arrow of time. This alternative is embedded in the Second Law of Thermodynamics. This determinant, which becomes clear once the thermodynamics of antimatter is considered theoretically[10, 11], relates the flow of entropy to the order of events in time.

Key Words: Carnot cycle, gibbs equation, reversibility, second law of thermodynamics, symmetry, time direction

1. Introduction

One characteristic of Einstein's approach to physics was to assume that the correct form of the Laws of Nature would be formally symmetrical and that problems in physics could be identified and solved by discovering formal asymmetries in the fundamental theories and finding a way to make them symmetrical[12, 13, 14].

This is the first in a series of papers in which we make the Second Law of Thermodynamics formally symmetrical.

The dynamical equations of mechanics, which describe many aspects of the macroscopic and microscopic world given the stipulation that friction is negligible, are typically symmetrical with respect to the direction of time. They describe the ideal physical world in terms of reversible processes. By contrast, the equations of thermodynamics are concerned with the macroscopic description of phenomena and not with microscopic properties of matter. They describe reversible as well as irreversible processes. The Second Law of Thermodynamics defines reversible processes as those in which the change of entropy is equal to zero, and irreversible processes as those in which there is a net increase of entropy[15]. This is the essential distinction between reversible and irreversible processes. It is widely held that irreversible processes as opposed to reversible processes are not invariant with respect to the direction of time[16, 17, 18, 19, 20, 21], and the increase of entropy that occurs in irreversible processes serves as a criterion to distinguish past and future time. Eddington[22] concluded that "time's arrow," which is also known as the direction of "proper time," points in the direction of the increase of entropy. However, analysis of a reversible cyclical thermal energy converter, in which thermal energy is converted into work, has shown us that, in theory, there is an asymmetry in the events occurring on the time axis, even in reversible thermodynamic systems where there is no net change in the total entropy. This leads us to the hypothesis that another thermodynamic arrow of time might be established by means of a second determinant which is more fundamental than the increase of entropy in irreversible processes. This determinant, which we will consider in the following paper, and which becomes clear once the thermodynamics of matter and antimatter are considered theoretically[11], is not explicit but implicit in the current form of the Second Law of Thermodynamics. It is explicit in the time symmetrical Second Law of Thermodynamics[11]. The time symmetrical law does not describe the symmetry of matter moving forward or backwards in time, which is known as T-symmetry or time reversal symmetry, but rather it describes the symmetry of the order of events in time that are experienced by matter compared with antimatter when challenged with an identical force[10]. We emphasize that the Second Law of Thermodynamics and the thermodynamic arrow of time describe the order of events in time, and not time itself[23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

2. Results and conclusions

A thermal energy converter is a device that converts thermal energy to another form of energy (mechanical, chemical, electrical, magnetic, etc.). The thermal energy converter we analyze hereafter is considered to be a closed system that transforms an input of thermal energy into any other form of energy without exchanging matter with the external world. A reversible process is defined as one in which the total entropy of the closed system and its surround does not change ($\oint dS_{total} = 0$). By contrast, an irreversible process is one that returns to its initial state with a net change in total entropy ($\oint dS_{total} > 0$). Planck[34] emphasized that the distinction between reversible and irreversible processes is crucial to the development of theoretical physics.

We will use the fundamental equation of thermodynamics[35] to define and analyze a thermal energy converter. This equation can be expressed either in terms of energy or entropy. When it is expressed in terms of energy, the internal energy, U, is a function of all the extensive parameters of the system as follows:

$$U = U(S, V, N_i, X_k) \tag{1}$$

where S is the entropy, V the volume, N_i the number of moles of the ith kind of matter, and X_k is any

extensive parameter of the kth type (e.g. charge or mass). The complete differential of Equ. (1) is:

$$dU = (\partial U/\partial S)_{V,N_i,X_k} dS + (\partial U/\partial V)_{S,N_i,X_k} dV + (\partial U/\partial N_i)_{S,V,X_k} dN_i + (\partial U/\partial X_k)_{S,V,N_i} dX_k \tag{2}$$

The partial derivatives are defined as the intensive parameters, and they correspond to the known intensive parameters of classical thermodynamics[36]. $(\partial U/\partial S)_{V,N_i,X_k} \equiv T$, the absolute temperature; $(\partial U/\partial V)_{S,N_i,X_k} \equiv -P$, the negative of the hydrostatic pressure; $(\partial U/\partial N_i)_{S,V,X_k} \equiv \mu_i$, the chemical potential of the ith kind of matter, and $(\partial U/\partial X_k)_{S,V,N_i} \equiv Y_k$, any other intensive parameter (e.g. electrical or gravitational potential). Thus

$$dU = TdS + (-P)dV + \mu_i dn_i + Y_k dX_k \tag{3}$$

This is the Gibbs' equation, which is the complete differential of the fundamental equation of thermodynamics[36, 37]. In this equation, the entropy (S) appears as a state function and TdS represents the thermal energy. Thus the Gibbs equation can be seen as the generalized macroscopic law of energy conservation.

Every point in the multidimensional space defined by U, S, V, N_i, X_k represents an equilibrium state, and any line in the multidimensional space represents a continuous sequence of equilibrium states, known as a "quasi-static process"[35, 38]. We consider the quasi-static process to be an ordered sequence of events in the time domain, but not necessarily a temporal sequence of equidurational events, or any other definite analytic form. Again we emphasize that we are considering the order of events in time and not time itself.

The entropy function, S, which is at the center of the formulation of the Second Law of Thermodynamics is quite an unusual function in physics. Reversibility imposes the constraint that the entropy be a total differential, i.e. a conserved quantity like all the other extensive parameters, which are state functions, in the Gibbs equation. It is only in the mode of irreversible changes that the entropy function "enjoys" a unique status, having a dual role: as the extensive parameter of the thermal energy term of the internal energy and also as a criterion of the irreversibility of a given change, in which case S is not a conserved quantity. In this respect, entropy is unique when compared with all other extensive parameters in irreversible processes[39].

The thermal energy converter used in our analysis is a closed system that has access to two separate reservoirs of thermal energy (TdS), each at a different temperature (h=hot and c=cold; i.e. $T_h > T_c$) as well as a work reservoir[35] in which energy is stored in a non-thermal state. The need for at least two thermal energy reservoirs at different temperatures to produce useful work in a cyclical energy converter follows directly from the formulation of the Second Law of Thermodynamics by Lord Kelvin[40, 41]: "Any closed system which undergoes an isothermal cyclic process cannot produce positive work."

For the analysis of a reversible cyclical thermal energy converter, we have to integrate Equ. (3) around the complete cycle:

$$\oint dU = \oint TdS - \oint PdV + \oint \mu_i dN_i + \oint Y_k dX_k \tag{4}$$

At least two terms from the right side of Equ. (4) are needed for the description of any cyclical reversible thermal energy converter. One is the input term, which is $\oint TdS$ (= thermal energy or heat). The second is the output term, $\oint Y_k dX_k$, which can be any other term of the right hand side of Equ. (4), including $\oint PdV$ and $\oint \mu_i dN_i$ but not $\oint TdS$, the input term. $Y_k dX_k$ represents the non-thermal energy that will eventually be stored in the work reservoir.

For our analysis, we will perform a geometrical analysis[36, 42] using the thermal energy plane (TS) to describe the operation of a reversible cyclical thermal energy converter known as the Carnot cycle[43] operated quasi-statically.

The terms in Equ. (4), in which the intensive parameter is held constant, vanish because the constant intensive parameters come out of the integrals and all the extensive parameters, including U, the internal energy, are state functions, i.e. perfect differentials which vanish after integration. In our analysis, all the intensive parameters in Equ. (4) except T and Y_k will be considered constant. Thus

$$\oint dU = \oint TdS + \oint Y_k dX_k = 0 \tag{5}$$

and

$$\oint TdS = - \oint Y_k dX_k \tag{6}$$

This is the equation of the Carnot cycle of the thermal energy converter, in which the thermal energy term, $\oint TdS$, is the input term and $\oint Y_k dX_k$ is the work output, which will appear in the work reservoir eventually when the cycle is completed. Equ. (6) also expresses the geometrical fact that the area of the thermal energy input on the thermal energy plane (TS) will be equal to that of the output work on the appropriate plane (YX). The input term $\oint TdS$ and the output term $\oint Y_k dX_k$ have opposite signs. When the cycle is operated to lead to the production of useful work, then by definition:

$$\oint Y_k dX_k < 0 \tag{7}$$

and thus

$$\oint TdS > 0 \tag{8}$$

Since the input and output terms are equal in magnitude, it is sufficient to integrate only one of them. The output term will typically be more difficult to integrate because of the functional relationship between the extensive parameter X_k and its conjugate intensive parameter Y_k. On the other hand, for the conditions of the operation of a Carnot cycle (two isotherms and two adiabatics), the integration of $\oint TdS$ is straightforward and simple.

The Carnot cycle is composed of four "strokes" on the thermal energy plane. Two of the strokes are isothermal and the other two are adiabatic. On the thermal energy plane, the isotherms are straight lines parallel to the abscissa, and the adiabatics turn out to be straight lines parallel to the ordinate[36]. As can be seen in Figure 1, under all conditions:

$$S_2 > S_1$$
$$S_3 > S_4$$
$$S_1 = S_4$$
$$S_2 = S_3$$

The integration of the input term in Equ. (6) is:

$$\oint TdS = T_h \int_1^2 dS + \int_2^3 TdS + T_c \int_3^4 dS + \int_4^1 TdS \tag{9}$$

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The two adiabatic integrals vanish, as they are held at isentropic conditions (i.e. $S =$ constant). Therefore, Equ. (9) is reduced to:

$$\oint T dS = T_h \int_1^2 dS + T_c \int_3^4 dS \tag{10}$$

Mathematically, the cycle of the thermal energy converter can be operated in two directions, clockwise and anticlockwise. When integrated clockwise we find:

$$\oint T dS = T_h(S_2 - S_1) + T_c(S_4 - S_3) \tag{11}$$

Rearranging Equ. (11) we get:

$$\oint T dS = T_h(S_2 - S_1) - T_c(S_3 - S_4) \tag{12}$$

and since $(S_2 - S_1) = (S_3 - S_4)$, it follows that

$$\oint T dS = (T_h - T_c)(S_2 - S_1) \tag{13}$$

Since $(T_h - T_c) > 0$ and $(S_2 - S_1) > 0$, then $\oint T dS > 0$ and $\oint Y_k dX_k < 0$ and thus useful work can be obtained from the operation of the cycle. When integrating anticlockwise we find:

$$\oint T dS = T_h(S_1 - S_2) + T_c(S_3 - S_4) \tag{14}$$

Rearranging Equ. (14) we get:

$$\oint T dS = T_c(S_3 - S_4) - T_h(S_2 - S_1) \tag{15}$$

and since $(S_2 - S_1) = (S_3 - S_4)$, it follows that:

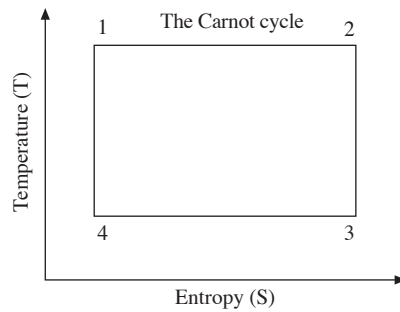$$\oint T dS = (T_c - T_h)(S_2 - S_1) \tag{16}$$

Figure 1. Diagram of the Carnot cycle of a thermal energy converter presented geometrically on the temperature-entropy plane. The two isotherms are parallel to the abscissa and the two adiabatics are parallel to the ordinate. Mathematically, the thermal energy term, $\oint T dS$, can be integrated clockwise or anticlockwise.

Since $(T_c - T_h) < 0$ and $(S_2 - S_1) > 0$, then $\oint T dS < 0$ and $\oint Y_k dX_k > 0$. Thus it is impossible to obtain useful work from the operation of the cycle in the anticlockwise direction.

This asymmetry in the thermal energy converter is physically based on the fact that the isothermal input of thermal energy during the flow from the hot reservoir is greater than the isothermal output of thermal energy flowing into the cold reservoir. Consequently, more work is produced by the first and second stroke than is necessary to return the system to its initial state in the third and fourth stroke, and there is a conversion of thermal energy into useful work.

We have shown that the Carnot cycle can convert thermal energy to useful work only when it is operating in a clockwise direction, that is, when thermal energy flows from the hotter to the colder temperature. The lines of the cycle consist of points associated with the equilibrium states through which the system passes. These lines may be called the path of the system. The concept of path implies the idea of direction, i.e. direction in time, to express the order in which the system passes through the series of states [36]. Thus, we have seen that the direction of the thermal machine is asymmetric. Useful work can be obtained from the machine only when the path followed is in one of the two available directions. This is the arrow of time of the reversible thermal energy converter, which has nothing to do with an increase of entropy within the system.

One should note the distinction between the two available directions in which a thermal energy converter can theoretically run when it converts an input of thermal energy into useful work and the two available directions in which a traditional reversible thermal machine does run[35]. When a traditional thermal machine runs in one direction, it acts as a thermal energy converter and it converts an input of thermal energy into useful work. The traditional thermal machine can be made to run in the reverse direction by using an input of mechanical energy to pump thermal energy from a cold reservoir to a hot reservoir. The distinction between a thermal energy converter, which can theoretically run in one of two directions when it converts thermal energy into useful work; and a traditional thermal machine, which converts thermal energy into useful work when it runs in one direction and mechanical energy into thermal energy when it runs in the other direction is very important in regards to our discussion of the order of events on the time axis.

The Second Law demands that at the end of any number of iterations of a reversible cycle there be no change of entropy, either in the system or in the outer world. Reversibility, then, demands that there should be the same amount of entropy input (ΔS_h), an event that occurs during the isothermal contact with the hot reservoir, and entropy output (ΔS_c), an event that happens during the isothermal contact with the cold reservoir. Thus:

$$\Delta S_h + \Delta S_c = 0 \tag{17}$$

where $\Delta S = S_{final} - S_{initial}$ and represents the change of entropy in the hot or cold reservoirs, after and before a given number of cycles. Although it is not important for the present argument, it is important to note that the definitions of final time and initial time are independent of the direction of the cycle.

At the conclusion of the Carnot cycle, $\oint dS = 0$ and there is no change in the quantity of entropy in either the system or the surroundings. However, there is a net transport of entropy from the hot reservoir to the cold one, i.e. a new spatial distribution of entropy. The requirement that $\oint dS = 0$ for a reversible cycle necessitates that at least two kinds of energy participate, since

$$T_h \Delta S_h \neq -T_c \Delta S_c \tag{18}$$

But

$$T_h \Delta S_h + T_c \Delta S_c = - \oint Y_k dX_k \tag{19}$$

The term on the right hand side represents the output work. When we compare Equ. (18) and Equ. (19), we see the reason why there is a need to have at least two terms of the Gibbs equation to operate the reversible Carnot cycle. If both input and output energy terms were only thermal energy, it would be impossible to satisfy Equ. (18) and Equ. (19) without the coupled new kind of energy. Thus the participation of another kind of energy ($Y_k dX_k$) that can be coupled quasi-statically to the thermal energy is a necessary condition for a reversible thermal energy converter. The operation of a cycle in a reversible thermal energy converter results in a spatial redistribution of entropy from the high temperature reservoir to the low temperature reservoir without any dissipation of energy or increase in entropy in the system and the surroundings.

Interestingly, since TdS and $Y_k dX_k$ are not complete differentials, these functions wind out of the two-dimensional TS or YX plane, respectively. While the third dimension is not explicitly defined by the Gibbs equation, it is reasonable to conclude that it represents a time axis[44] with a defined direction but an undefined scale. The explicit definition of the third axis of time serves to emphasize that changes in the state of the system cannot be instantaneous[5, 6, 7, 8, 9]. The axis of time, however, indicates the direction of change, though not its rate. When the third axis is defined as the time axis, the three-dimensional volume enclosed by the functions represents the action (energy x time) as defined by Kelvin and Tait[45] and extended by Planck[46] when he defined h as the quantum of action.

In conclusion, in analyzing the sequence of events that occur in the time domain of cyclical reversible energy converters, we have shown that there is a preferred order of events in the time domain when there is no change in the magnitude of entropy. By contrast, the fundamental dynamical equations in physics, which are formally second-order with respect to time as a result of the assumption that friction is negligible, show no asymmetry with respect to the order of events in time[34, 47, 48]. In the following paper[11], in which we model the thermodynamics of antimatter, we present a way to mend the asymmetry we found in the Second Law of Thermodynamics that was developed exclusively from the study of matter[10]. Consequently, the thermodynamic equations, like the wave equation[13, 14], which describes the movement of energy and momentum from a source to a sink, will be symmetrical with respect to the order of events along the time axis.

Acknowledgements

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